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# Composite washing of coals from multiple sources: Optimization by numerical technique

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**Abstract**

Optimization of operating parameters for achieving maximum yield of cleans at the desired ash level in composite coal washeries is a major problem, particularly when the washeries receive raw coals from a large number of mines and there is a frequent variation in the proportion of coals from different sources or in their individual characteristics. This problem is more acute in countries with a predominant reserve of difficult-to-wash coals. Control of parameters in such cases requires too frequent sampling and characterization of the blended feed as the graphical method for parameter optimization based on individual coal characterization is not practicable for large numbers of source coals or washing size fractions. This paper presents a solution in the form of a numerical method with any number of source coals or washing size fractions, with the help of constrained non-linear optimization of a multivariable objective function. Operational constraints of different washers in the composite plant functioning on the principle of gravity separation can also be largely overcome as any variation in the elementary ash is incorporated in the method in the form of a set of inequality constraints along with the range of (available) specific gravity of cuts and desired ash content of the overall cleans. A case study with nine coals of different characteristics under two different mode of operations is presented.

## 1. Introduction

In composite washeries, the raw coals received are blended after preliminary crushing and then screened into several size fractions, and each size fraction is treated separately in suitable washing circuits operating on parameters determined on the basis of laboratory float and sink tests. Optimization of operating parameters for achieving maximum yield at the desired ash level in different washers is based on the determination of cleaning characteristics of the respective

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feed. However frequent variation in the blend render control of parameters difficult.

In the graphical method (Sarkar et al., 1960) for maximization of yield of composite cleans at the desired ash content it was suggested that cleaning of coarser size fractions with a poorer washing characteristics should be carried out at a slightly higher ash level, and that of smaller size fractions with better washing characteristics at a lower ash level. The study revealed that the method could achieve an increase of 3.5% in the yield of cleans, compared to washing the coarser and smaller size fractions each at the desired ash content. It was also observed that coals of dissimilar washing characteristics could be blended without any loss of yield of composite cleans. However, the graphical method could not be of much practical use for a large number of feed coals or washing size fractions as it involved a very elaborate calculation and intricate interpolations. Thus, frequent sampling and characterization of the blended feed in tune with the change in the proportion of coals from different sources had to be resorted to for parameter optimization in composite washeries.

A similar study on the problem of yield optimization in coal washing and blending (Salama, 1986) revealed the limitation of the graphical technique. Establishing that equalization of elementary ash of the coals at the respective specific gravity of cut is necessary for yield maximization at desired ash content, he advocated a numerical approach which is essentially a constrained nonlinear optimization problem.

However, in Salama's approach, the number of the explicit variables, the specific gravity (sp.gr.) of cut for individual coals $\delta_i$ (where $i=1,2,...,n$, $n$ being the number of coals) were replaced by a single variable $\lambda$, the elementary ash content, taken to be identical at all cut points, and the numerical search was made in that direction. This approach reduced the complexity of the multi-variable search problem but suffered from the limitation that it could not take into account any possible restrictions imposed on the available range of sp.gr. of cut due to operational constraints, when the necessary condition of elementary ash equalization might not be feasible.

The above limitation is overcome in the present study by utilizing a multivariable search technique of the explicit variable $\delta_{ij}$ ($i=1,2,...,n$ is the number of coals and $j=1,2,...,m$ is the number of washing size fractions) where the condition of identical elementary ash is replaced by a set of inequality constraints.

## 2. Basic concepts

Let $y(\delta)\mathrm{d}\delta$ be the mass fraction (in percent) of the coal within a specific gravity fraction, $\delta$ to $\delta+\mathrm{d}\delta$ and $\lambda(\delta)$ be the elementary ash at specific gravity, $\delta$. The cumulative mass (in percent) can be expressed as,

$$Y(\delta)=\int_{\delta_m}^{\delta} y(\bar{\delta})\mathrm{d}\bar{\delta} \tag{1}$$

where $\bar{\delta}$ is a dummy variable and $\delta_m$ is the density of the lightest particle present in the coal.

The cumulative ash can be expressed as,

$$A(\delta)=\int_{\delta_m}^{\delta}\lambda(\bar{\delta})\cdot y(\bar{\delta})\,d\bar{\delta}/Y(\delta) \tag{2}$$

It follows from Eqs. (1) and (2)

$$\frac{d}{d\delta}Y(\delta)=y(\delta)$$

$$\frac{d}{d\delta}\{A(\delta)\cdot Y(\delta)\}=\lambda(\delta)\cdot y(\delta)$$

$$\lambda(\delta)\cdot\frac{d}{d\delta}Y(\delta)=\frac{d}{dY(\delta)}\{Y(\delta)\cdot A(\delta)\}\cdot\frac{d}{d\delta}Y(\delta)$$

$$\lambda(\delta)=\frac{d}{dY(\delta)}\{Y(\delta)\cdot A(\delta)\} \tag{3}$$

## 3. Mathematical formulation

Let there be $n$ different coals with $m$ different size fractions (Figs. 1 and 2). The overall yield, $Y_{ld}$, can be expressed as,

$$Y_{ld}(\delta_1 \ldots \delta_j)=\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}R_i\cdot W_{ij}\cdot Y1_{ij}}{\sum_{j=1}^{m}\sum_{i=1}^{n}R_i\cdot W_{ij}} \tag{4}$$

The overall cumulative ash, $TA$, is similarly expressed as,

$$TA(\delta_1 \ldots \delta_j)=\frac{\sum_{j=1}^{m}\sum_{i=1}^{n}R_i\cdot W_{ij}\cdot Y2_{ij}}{\sum_{j=1}^{m}\sum_{i=1}^{n}R_i\cdot W_{ij}\cdot Y1_{ij}} \tag{5}$$

where:

$R_i$ = ratio of the $i$th coal in the feed mix

$W_{ij}$ = percent weight in the $j$th size fraction of the $i$th coal as determined by the screen analysis of individual coals expressed as percent of its feed to the composite washing system

$W_{ij}$ = Cumulative yield of $i$th coal in $j$th size fraction

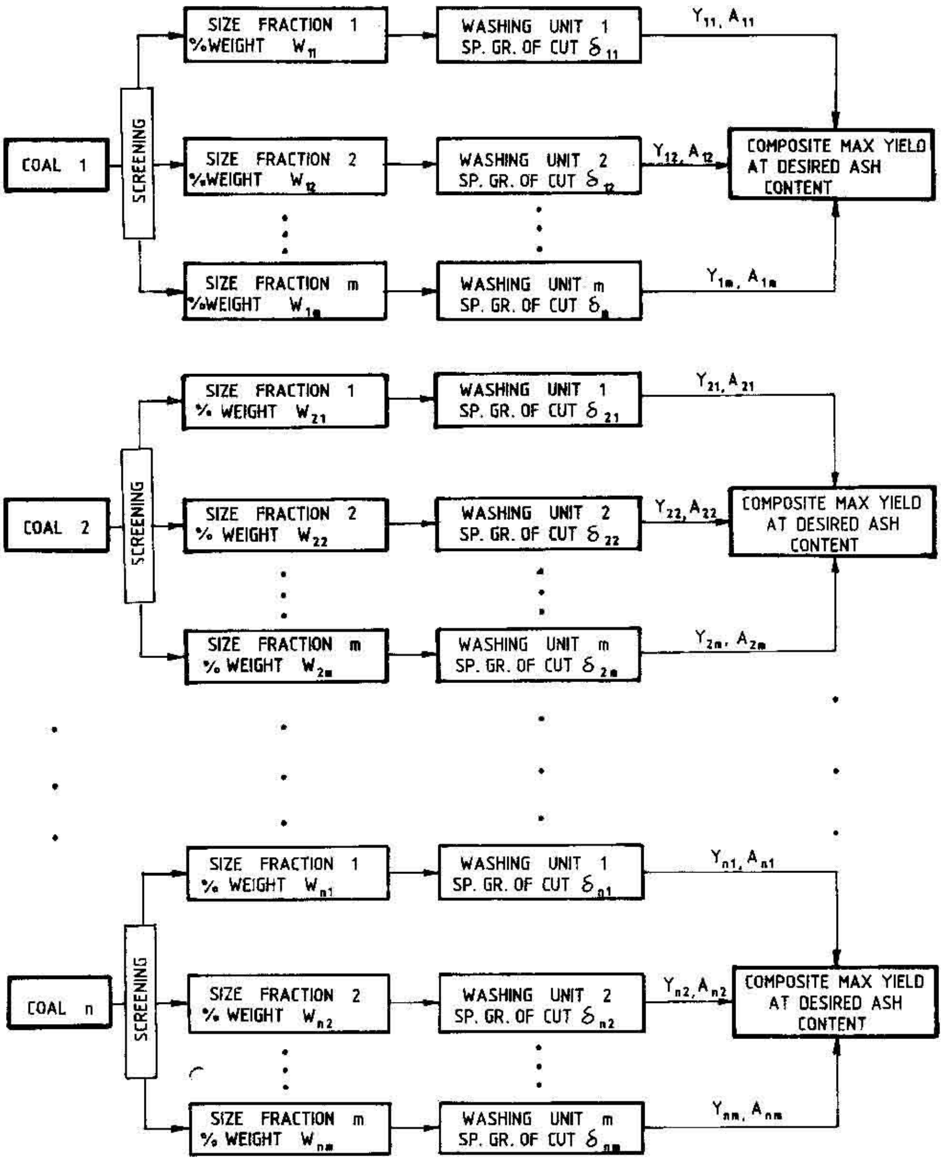


Fig. 1. Optimization in composite washing of feed coals individually.

$Y1_{ij}$ = Cumulative yield × cumulative ash of $i$th coal in $j$th size fraction

$$= Y1_{ij} \cdot A_{ij} \tag{6}$$

$A_{ij}$ = Cumulative ash of the $i$th coal of $j$th size fraction.

Eqs. (4) and (5) can be reduced to

$$Y_{\mathrm{ld}} = \frac{\sum_{j=1}^{m} C_j(\delta_j)}{\sum_{i=1}^{n} R_i} \tag{7}$$

$$TA = \frac{\sum_{j=1}^{m} D_j(\delta_j)}{\sum_{j=1}^{m} C_j(\delta_j)} \tag{8}$$

$$\lambda_j = \frac{\sum_{j=1}^{m} R_i \cdot W_{ij} \cdot \lambda_{ij}}{\sum_{i=1}^{n} R_i \cdot W_{ij}} \tag{9}$$

where:

$C_j(\delta_j)$ = Total cumulative yield in the $j$th size fraction at specific gravity $\delta_j$

$$D_j(\delta_j) = C_j(\delta_j) \cdot A_j(\delta_j) \tag{10}$$

$A_j(\delta_j)$ = Cumulative ash of all coals in the $j$th size fraction at specific gravity $\delta_j$
$\lambda_{ij}$ = elementary ash of $i$th coal in $j$th size fraction
$\lambda_j$ = Overall elementary ash of the $j$th size fraction

As the different feed coals are blended, the optimization problem is to determine the specific gravity cut points of each size fraction $\delta_j$, so as to maximize the overall yield at the desired ash level.

Mathematically it can be stated as:

$$\text{Maximize } Y_{\mathrm{ld}}(\delta_1 \ldots \delta_j) \tag{11}$$

subject to the constraint

$$TA(\delta_1 \ldots \delta_j) = b \tag{12}$$

where $b$ is the desired ash content.

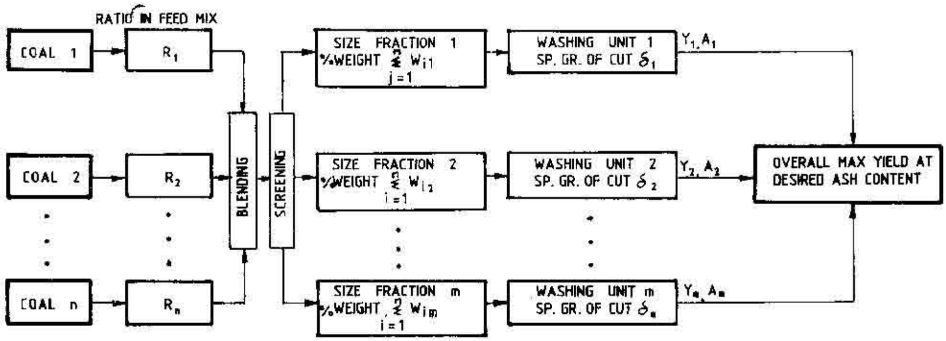


Fig. 2. Global optimization in composite washing of mixed coals.

## 4. Necessary condition for optimization

The necessary condition can be obtained with the help of a Lagrangian $\mathscr{L}$,

$$\mathscr{L}(\delta_1 ... \delta_j) = Y_{1d}(\delta_1 ... \delta_j) + \alpha [TA(\delta_1 ... \delta_j) - b] \tag{13}$$

where $\alpha$ is the Lagrangian multiplier.

Differentiating $\mathscr{L}$ w.r.t. $\delta_j$ and $\alpha$ and equating them to zero the necessary condition for optimal solution can be obtained. Thus,

$$TA(\delta_1^* ... \delta_j^*) = b \tag{14}$$

where $\delta_j^*$'s are the optimum cut points.

$$\frac{1}{\sum_{i=1}^{n} R_i} \cdot \frac{\mathrm{d}}{\mathrm{d}\delta_j^*} C_j + \frac{\alpha}{(\sum C_j)^2} \cdot \left[ \sum C_j \cdot \frac{\mathrm{d}D_j}{\mathrm{d}\delta_j^*} - \sum_j^D \cdot \frac{\mathrm{d}C_j}{\mathrm{d}\delta_j^*} \right] = 0$$

$$\frac{1}{\sum_{i=1}^{n} R_i} \cdot \frac{\mathrm{d}}{\mathrm{d}\delta_j^*} C_j = \frac{\alpha}{(\sum C_j)^2} \cdot \left[ \sum D_j \cdot \frac{\mathrm{d}}{\mathrm{d}\delta_j^*} C_j - \sum C_j \cdot \frac{\mathrm{d}}{\mathrm{d}C_j^*} D_j \cdot \frac{\mathrm{d}C_j}{\mathrm{d}\delta_j^*} \right]$$

$$\frac{1}{\sum_{i=1}^{n} R_i} = \frac{\alpha}{(\sum C_j)^2} \cdot \left[ \sum D_j - (\sum C_j) \frac{\mathrm{d}D_j}{\mathrm{d}C_j} \right] \tag{15}$$

It follows from the above that

$$\frac{\mathrm{d}D_j}{\mathrm{d}C_j} = \frac{\mathrm{d}D_{j+1}}{\mathrm{d}C_{j+1}} \quad \text{where} \quad j = 1 ... m$$

or $$\frac{\mathrm{d}(A_j \cdot C_j)}{\mathrm{d}C_j} = \frac{\mathrm{d}(A_{j+1} \cdot C_{j+1})}{\mathrm{d}C_{j+1}}$$

or $$\lambda_j(\delta_j^*) = \lambda_{j+1}(\delta_{j+1}^*) \tag{16}$$

Thus the necessary condition for maximization of yield at desired ash level is equalization of elementary ash.

## 5. Methodology

Polynomial equations up to a maximum of third degree are fitted to $Y1_{ij}$, $Y2_{ij}$ and $\lambda_{ij}$ of 9 Indian coals in the sp.gr. range 1.40 to 1.80, the generalised form being:

$$\lambda_{ij} \text{ or } Yr_{ij} = a_0 + \sum_{k=1}^{l} a_k \cdot \delta_{ij}^k \tag{17}$$

where $l$ is the degree of the equation, $r = 1$ denotes the equation for cumulative yield and $r=2$ the equation for cumulative yield × cumulative ash, where $\delta_{ij}$ is the specific gravity.

The best values of the model parameters ($a_0$, $a_1$, $a_2$, $a_3$) for cumulative yield, the product of cumulative yield and ash, and elementary ash of cleans are estimated for different coals and size fraction using non-linear regression by means of (Marquardt, 1963) algorithm and the computation is terminated when the residual changes ($RL$) in the function ($\phi$) satisfied the relation

$$RL\phi = \text{abs}[\{\phi^{t+1} - \phi^{t}\}/\{\phi^{t} + 10^{15}\}] \leqslant 10^{-10} \tag{18}$$

where $t$ indicates the iteration number and $\phi$ is the least square function, i.e. the summation of the square of the difference between the calculated and experimental values from float and sink experiments over the total number of sp.gr. of cuts.

The degree of polynomial regression is decided in individual cases on the basis of statistical reliability tests (0.95 probability level). For individual coefficients, "$t$" values are calculated. In case the "$t$" values are found to be insignificant for any paramenters, the same are eliminated from the equation and the regression coefficients are recalculated.

In the proposed method the objective function, Eq. (11), is being maximized subject to the equality constraint, Eq. (12) and the set of inequality constraints given by the following equation:

$$-e \leqslant (\lambda_j - \lambda_k \leqslant +e \tag{19}$$

where $j=1,2,...,(m-1)$, $k=j+1,j+2,...,m$, and $e=0.1$ or allowable difference in elementary ash values at the cut points.

The criteria for elementary ash equalization i.e. at optimum cut points is replaced by the inequality constraints (Eq. 19, total number in the set being combination ${}^{m}C_2$) with the objective of accommodating various restrictions on the lower and upper bounds of the variables $\delta$ (sp.gr.) when the condition of ash equalization may not be valid. In the case where ash equalization is necessary, the value of $e$ is taken as 0.1, which is within the error allowable in determination of ash contents. This helps in rapid convergence to global maximum and gives results identical to that obtained by searching along the direction of elementary ash, where ash equalization is implied. However, when further restrictions on the sp.gr. of cut due to operational constraints are required to be imposed, the same computer programme may be used by suitably increasing the value of $e$ in Eq. (19) for achieving optimum results under the constraints.

The maximum value of $Y_{1d}$ is computed, keeping in view the restrictions imposed, as mentioned above, using the "automatic" method (Rosenbrock, 1960; Rosenbrock and Storey, 1966).

Table 1
Washability equations of feed coals

| Coal Overall ash (%) | Size (mm) wt (%) ash (%) | Function (%) | Model parameters (sp.gr. range 1.40–1.80) | | | |
|---|---|---|---|---|---|---|
| | | | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
| A | 75–6 | $Y$ | −1274.1 | 1486.9 | −403.2 | – |
| 28.4 | 90.25 | $Y{\cdot}A$ | 1527.5 | −3095.6 | 2051.5 | −441.6 |
| | 29.6 | $\lambda$ | −98.4 | 83.8 | – | – |
| | 6–0.5 | $Y$ | −791.8 | 972.1 | −267.5 | – |
| | 9.75 | $Y{\cdot}A$ | −144.5 | 159.4 | −38.8 | – |
| | 22.0 | $\lambda$ | −106.1 | 87.7 | – | – |
| B | 75–6 | $Y$ | −1196.3 | 1512.2 | −440.4 | – |
| 16.0 | 87.34 | $Y{\cdot}A$ | −293.2 | 358.6 | −103.6 | – |
| | 17.0 | $\lambda$ | −78.5 | 68.6 | – | – |
| | 6–0.5 | $Y$ | −310.6 | 459.9 | −129.8 | – |
| | 12.66 | $Y{\cdot}A$ | −63.7 | 76.9 | −20.3 | – |
| | 10.5 | $\lambda$ | −98.4 | 80.8 | – | – |
| C | 75–6 | $Y$ | −7705.7 | 13769.0 | −8121.3 | 1599.5 |
| 21.4 | 79.89 | $Y{\cdot}A$ | −1258.9 | 2196.7 | −1268.0 | 245.7 |
| | 22.9 | $\lambda$ | −92.4 | 79.1 | – | – |
| | 6–0.5 | $Y$ | −2609.4 | 4674.7 | −2709.5 | 524.8 |
| | 20.11 | $Y{\cdot}A$ | −102.1 | 122.9 | −34.0 | – |
| | 17.8 | $\lambda$ | −115.0 | 93.1 | – | – |
| D | 75–6 | $Y$ | −5544.7 | 9645.0 | −5528.9 | 1060.5 |
| 23.9 | 88.07 | $Y{\cdot}A$ | −219.1 | 255.9 | −69.1 | – |
| | 25.4 | $\lambda$ | −99.3 | 82.3 | – | – |
| | 6–0.5 | $Y$ | −684.1 | 893.8 | −256.2 | – |
| | 11.93 | $Y{\cdot}A$ | −143.5 | 173.4 | −48.1 | – |
| | 15.8 | $\lambda$ | −99.5 | 82.5 | – | – |
| E | 75–6 | $Y$ | −8807.7 | 15713.0 | −9302.2 | 1844.1 |
| 27.5 | 90.11 | $Y{\cdot}A$ | −1330.7 | 2345.7 | −1380.6 | 275.2 |
| | 28.7 | $\lambda$ | −102.4 | 85.7 | – | – |
| | 6–0.5 | $Y$ | −3277.5 | 5760.4 | −3307.4 | 636.4 |
| | 9.89 | $Y{\cdot}A$ | −134.6 | 155.7 | −41.1 | – |
| | 20.8 | $\lambda$ | −106.3 | 87.4 | – | – |
| F | 13–0.5 | $Y$ | −797.4 | 919.2 | −237.9 | – |
| 28.8 | 100.00 | $Y{\cdot}A$ | 104.1 | 96.3 | −14.6 | – |
| | 28.8 | $\lambda$ | −100.3 | 84.0 | – | – |

| Coal<br>Overall<br>ash (%) | Size (mm)<br>wt (%)<br>ash (%) | Function<br>(%) | Model parameters (sp.gr. range 1.40–1.80) | | | |
|---|---|---|---|---|---|---|
| | | | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
| G | 75–6 | $Y$ | −1830.8 | 2154.4 | −602.1 | – |
| 30.8 | 90.1 | $Y \cdot A$ | −416.4 | 472.5 | −124.9 | – |
| | 31.4 | $\lambda$ | −96.9 | 83.7 | – | – |
| | 6–0.5 | $Y$ | – | −481.0 | 588.1 | −163.0 |
| | 9.9 | $Y \cdot A$ | 921.6 | −1829.0 | 1185.8 | −249.1 |
| | 25.3 | $\lambda$ | −101.6 | 83.9 | – | – |
| H | 75–6 | $Y$ | −3954.0 | 6578.1 | −3617.4 | 671.3 |
| 35.4 | 93.46 | $Y \cdot A$ | −225.6 | 242.5 | −57.6 | – |
| | 36.2 | $\lambda$ | −97.2 | 83.4 | – | – |
| | 6–0.5 | $Y$ | −774.9 | 941.9 | −257.1 | – |
| | 6.54 | $Y \cdot A$ | −128.9 | 138.9 | −32.1 | – |
| | 24.7 | $\lambda$ | −112.0 | 91.3 | – | – |
| I | 75–6 | $Y$ | −630.9 | 660.1 | −145.8 | – |
| 35.3 | 88.54 | $Y \cdot A$ | – | −518.2 | 37.5 | – |
| | 36.3 | $\lambda$ | −104.8 | 88.3 | – | – |
| | 6–0.5 | $Y$ | −358.5 | 400.2 | −86.3 | – |
| | 11.46 | $Y \cdot A$ | 10.8 | −43.4 | 26.6 | – |
| | 29.6 | $\lambda$ | −112.0 | 91.7 | – | – |

$Y$=cum. yield; $A$=cum. ash; $\lambda$=elementary ash.

## 6. Case study

Three different problems are studied in this work. In the first case, each individual seam coal is washed in two size fractions (75–6 and 6–0.5 mm) only to produce maximum possible yield of composite cleans at desired ash content. In the second case the feed coals are blended in any predetermined ratio and then screened, and washed in the same two screen fractions under optimum conditions. The generalised schematic diagrams for the case (1) and case (2) are shown in Figs. 1 and 2 respectively. A few particular variations of case (2) with constrained operational parameters have been dealt with subsequently.

## 7. Results and discussion

Table 1 presents the overall ash of individual coals, weight percentages of washing size fractions 75–6 mm (coarser) and 6–0.5 mm (smaller) along with ash content and the values of regression parameters corresponding to the model Eq. (17), in the specified sp.gr. range. In all cases the correlation coefficients are found to be more than 0.99.

Table 2 presents the optimum yield, sp.gr. of cut, elementary ash at cut point

Table 2
Composite washing of individual coals

| Sl. No. | Coal | Operable sp.gr. of cut | | | | Allowable diff. in elem. ash (%) at cut pt. | Optimum sp.gr. of cut | | Elementary ash (%) at cut point | | Optimum cleans (%) | | | | Desired composite clean (%) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 75–6 mm | | 6–0.5 mm | | | | | | | 75–6 mm | | 6–0.5 mm | | | |
| | | Low | High | Low | High | | 75–6 mm | 6–0.5 mm | 75–6 mm | 6–0.5 mm | Yield | Ash | Yield | Ash | Yield | Ash |
| 1 | A | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.461 | 1.484 | 24.1 | 24.0 | 37.8 | 18.1 | 61.5 | 10.6 | 40.1 | 17.0 |
| 2 | B | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.592 | 1.597 | 30.7 | 30.7 | 95.1 | 16.0 | 92.9 | 7.8 | 94.8 | 15.0[a] |
| 3 | C | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.621 | 1.622 | 35.9 | 35.9 | 88.0 | 18.9 | 84.1 | 9.3 | 87.2 | 17.0 |
| 4 | D | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.612 | 1.610 | 33.3 | 33.2 | 78.3 | 17.8 | 90.9 | 12.1 | 79.8 | 17.0 |
| 5 | E | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.483 | 1.498 | 24.6 | 24.6 | 50.9 | 18.1 | 69.3 | 9.4 | 52.7 | 17.0 |
| 6 | F | – | – | 1.45 | 1.65 | – | – | 1.516 | – | 27.0 | – | – | 49.5 | 17.0 | 49.5 | 17.0[b] |
| 7 | G | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.422 | 1.474 | 22.0 | 22.1 | 15.0 | 19.7 | 45.6 | 8.8 | 18.0 | 17.0 |
| 8 | H | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.428 | 1.464 | 21.5 | 21.5 | 17.5 | 18.3 | 53.1 | 10.8 | 19.9 | 17.0 |
| 9 | I | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.454 | 1.478 | 23.5 | 23.5 | 20.7 | 18.8 | 44.5 | 10.6 | 23.4 | 17.0 |
| Average | | 1.40 | 1.65 | 1.45 | 1.65 | 0.1 | 1.509 | 1.527 | 27.0 | 26.8 | 50.4 | 18.2 | 65.7 | 10.7 | 51.7 | 16.8 |

[a] Raw coal ash was 16.0%.
[b] Coal was crushed to – 13 mm and considered for washing with the smaller fractions.

Table 3
Global optimiztion of mixed coals

| Sl. No. | Coal | Ratio in feed mix | Operable sp.gr. of cut | | | | Allowable diff. in elem. ash (%) at cut pt. | Optimum sp.gr. of cut | | Elementary ash (%) at cut point | | Optimum cleans (%) | | | | Desired composite clean (%) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 75–6 mm | | 6–0.5 mm | | | | | | | 75–6 mm | | 6–0.5 mm | | | |
| | | | Low | High | Low | High | | 75–6 mm | 6–0.5 mm | 75–6 mm | 6–0.5 mm | Yield | Ash | Yield | Ash | Yield | Ash |
| 1 | A | 1 | | | | | | | | 27.3 | 26.9 | 49.0 | 19.5 | 67.2 | 11.7 | 50.8 | 18.5 |
| | B | 1 | | | | | | | | 24.4 | 24.2 | 81.1 | 14.3 | 88.3 | 7.1 | 82.0 | 13.3 |
| | C | 1 | | | | | | | | 26.3 | 26.1 | 73.1 | 16.9 | 78.8 | 7.7 | 74.3 | 15.0 |
| | D | 1 | | | | | | | | 24.1 | 25.6 | 61.9 | 15.0 | 82.1 | 10.8 | 64.3 | 14.3 |
| | E | 1 | | | | | | | | 26.2 | 26.2 | 55.6 | 18.5 | 71.4 | 9.8 | 57.2 | 17.4 |
| | F | 1 | | | | | | | | – | 27.0 | – | – | 49.4 | 16.9 | 49.4 | 16.9 |
| | G | 1 | | | | | | | | 28.5 | 25.6 | 46.0 | 24.6 | 54.6 | 11.4 | 46.9 | 23.1 |
| | H | 1 | | | | | | | | 27.9 | 26.5 | 39.6 | 21.6 | 62.2 | 12.7 | 41.1 | 20.7 |
| | I | 1 | | | | | | | | 27.6 | 27.0 | 31.2 | 21.4 | 49.9 | 12.3 | 33.3 | 19.8 |
| | Overall | | 1.40 | 1.60 | 1.45 | 1.65 | 0.1 | 1.500 | 1.516 | 26.4 | 26.4 | 54.3 | 18.3 | 59.8 | 12.8 | 55.5 | 17.0 |
| 2 | A | 3.9 [a] | | | | | | | | 23.6 | 24.3 | 36.0 | 17.5 | 62.3 | 10.6 | 38.6 | 16.4 |
| | B | 2.1 | | | | | | | | 21.4 | 21.8 | 71.8 | 12.9 | 86.3 | 6.7 | 73.6 | 12.0 |
| | C | 2.2 | | | | | | | | 22.8 | 23.4 | 62.4 | 15.7 | 76.3 | 7.2 | 65.2 | 13.7 |
| | D | 4.0 | | | | | | | | 20.5 | 23.2 | 50.8 | 13.8 | 78.5 | 10.2 | 54.1 | 13.1 |
| | E | 11.7 | | | | | | | | 22.4 | 23.7 | 42.4 | 17.2 | 67.6 | 9.0 | 44.9 | 16.0 |
| | F | 30.6 | | | | | | | | – | 24.6 | – | – | 43.5 | 15.7 | 43.5 | 15.7 |
| | G | 21.7 | | | | | | | | 24.9 | 23.2 | 29.5 | 22.9 | 49.3 | 9.9 | 31.5 | 20.9 |
| | H | 21.0 | | | | | | | | 24.2 | 23.9 | 27.1 | 19.8 | 57.3 | 11.7 | 29.0 | 18.8 |
| | I | 2.3 | | | | | | | | 23.7 | 24.4 | 21.1 | 19.3 | 45.8 | 11.1 | 23.9 | 17.5 |
| | Overall | | 1.40 | 1.60 | 1.45 | 1.65 | 0.1 | 1.456 | 1.487 | 24.0 | 23.9 | 37.4 | 18.5 | 59.2 | 9.8 | 39.9 | 17.0 |

[a] As per ratio of geological reserve in particular seam.

Table 4
Global optimization of mixed coals [a] under constrained conditions

| Sl. No. | Operable sp.gr. of cut | | | | Allowable diff. in elem. ash (%) at cut pt. | Optimum sp.gr. of cut | | Elementary ash (%) at cut point | | Optimum cleans (%) | | | | Desired composite clean (%) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 75–6 mm | | 6–0.5 mm | | | 75–6 mm | 6–0.5 mm | 75–6 mm | 6–0.5 mm | 75–6 mm | | 6–0.5 mm | | | |
| | Low | High | Low | High | | | | | | Yield | Ash | Yield | Ash | Yield | Ash |
| 1 | 1.40 | 1.60 | 1.65 [b] | 1.80 | 20.0 | 1.403 | 1.650 | 19.3 | 38.1 | 14.1 | 12.8 | 73.1 | 19.8 | 36.1 | 18.1 |
| 2 | 1.40 | 1.60 | 1.60 [b] | 1.80 | 15.0 | 1.404 | 1.603 | 19.3 | 34.1 | 14.4 | 13.0 | 66.6 | 18.5 | 33.9 | 17.0 |
| 3 | 1.40 | 1.60 | 1.40 | 1.45 [b] | 5.0 | 1.473 | 1.449 | 25.1 | 21.2 | 40.0 | 19.6 | 38.5 | 12.4 | 39.4 | 17.0 |
| 4 | 1.40 | 1.45 [b] | 1.45 | 1.60 | 5.0 | 1.448 | 1.518 | 23.0 | 26.9 | 31.6 | 18.3 | 52.5 | 15.6 | 39.3 | 17.0 |

[a] Ratio as per geological reserve given in Table 3 (Sl. No. 2).
[b] Operational constraint.

for washing individual coal (case 1) to achieve 17% composite ash. It is observed that:

- the maximum composite yield at the desired ash level varies from 18.0 to 87.2% for individual coals;
- the desired sp.gr. of cut for coarser fraction varies from 1.422 to 1.621 and for smaller fraction varies from 1.464 to 1.622. The range of variation is too wide for practical operation;
- the average yield of cleans at about 17% ash assuming equal proportion of all the seam coals is 51.7%.

The global optimization of blended feed coals (case 2), assuming equal proportion in feed mix, shows that (Table 3):

- the yield of overall cleans is further increased by 3.8% at 17.0% ash content;
- the sp.gr. of cut for the coarser and smaller coal fractions are 1.500 and 1.516 respectively, which are within normal operating ranges of washeries.

It is observed that the inferior raw coals, viz. G, H and I contribute much higher amount of individual cleans in comparison to case 1 (Table 2). The yield of coarser fraction from coal G, which is as low as 15% in case 1 increases by three fold i.e. by 46.0% in case 2. Similarly, for coals H and I, the yield of the same size fraction increases from 17.5 to 39.6% and from 20.7 to 31.2% respectively. It is noted that the ash level of inferior coals increases but is compensated by the lower ash values of better washable coals, viz. B, D and C as shown in Table 3.

Considering blending of raw coals in the geological reserve proportion, the yield of cleans at 17.0% ash is 39.9% as shown in Table 3. As the proportion of poorer coals is higher, the optimum sp.gr. of cuts are required to be reduced to 1.456 and 1.487.

Results of optimization of the above blend under restricted operation of the washing plant are given in Table 4.

In serial No. 1, it is assumed that the small coal jig may not be operated at sp.gr. of cut less than 1.65. In this case the ash content of overall cleans can not be lowered to 17% under the given conditions. However, the overall cleans ash may still be maintained at 18.1% when the sp.gr. of cuts are 1.403 and 1.65, and the yield is 36.1%.

In serial No. 2, the same constraint is lowered to 1.60. The overall yield of cleans at 17% ash becomes 33.9%.

In serial No. 3, it is assumed that the dense medium cyclone for washing of smaller size can not be operated at sp.gr. of cut more than 1.45, due to constraints in the medium circuit. The coarser fraction cleaning should be carried out at 1.473 (instead of 1.45 as shown in Table 2) and the overall yield is 39.4%.

In case of restriction in the higher limit of sp.gr. of the coarser fraction as 1.45 (serial No. 4), the optimization programme suggests operation of the smaller fraction at 1.518 sp.gr. to yield overall cleans of 39.3% at 17% ash content. It may be noted that the allowable difference in elementary ash levels of different coals/ size fractions had to be increased for optimization under restricted operational conditions.

The successful application of the optimization methods, graphical or numerical, for maximizing composite yield at the desired ash level will however depend on accurate adjustments of parameters at the various unit operation systems and their efficiency of separation.

## 8. Conclusion

The numerical optimization method proposed in this study can effectively consider the wide variations in the proportion of raw coal mix from different feed coals, whose individual washability characteristic may be known or updated from time to time. The method can consider any number of feed coals or washing size fractions and can handle operational constraints on sp.gr. of cut.

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